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Predictive Processing, Rational Constructivism, and Bayesian Models of Development: Commentary

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Abstract

This is a commentary for a special issue on predictive processing and rational constructivist models of development. Mainly I use the opportunity to ask a bunch of questions about what these theoretical frameworks show us (and what they do not) and mostly where the open questions still are. To get meta for a moment, I thought these questions were the best way to maximize the value of my commentary: They have the highest probability of leading to the most uncertainty reduction for our field in the long term. Please read in that spirit.

Keywords: Bayesian models; Development; Optimality; Precision; Predictive processing; Rational constructivism

1. Introduction

Predictive processing (Clark, 2013; Friston, 2009) and rational constructivism (Gopnik & Wellman, 2012; Xu, 2019) are two of the most prominent explanatory frameworks in cognitive science, offering insights into multiple aspects of human behavior and cognition. Both overlap strongly with many aspects of the Bayesian framework, which suggests that human inferences can be profitably understood as a type of rational belief updating (Tenenbaum, Kemp, Griffiths, & Goodman, 2011). Having done some work in Bayesian modeling, when I

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was invited to give this commentary I said yes because I thought it would be a good opportunity to learn more about predictive processing and force myself to interrogate these different approaches—really figure out their similarities and differences, what they do well, and what they leave open.

Indeed, it was a pleasure to read the variety of papers in this special issue. I especially appreciated the tutorial by Sprevak and Smith (2024); it was one of the clearest and most comprehensive explanations of predictive processing I have read, with a perfect balance between intuitive explanations and inclusion of the mathematical details. From there, several articles focused on extending and testing predictive processing. Lazarova, Huang, Muckli, and Petro (2024) applied it to visual perception, demonstrating how perceptual priors and contextual feedback are combined when viewing ambiguous images, and Ciaunica, Levin, Rosas, and Friston (2024) offered an original perspective using it to illuminate the prenatal relationship between a mother's immune system and the developing child.

Multiple papers focused on important issues that specifically arise in development after birth. Andersen and Kiverstein (2024) ask why children play and suggest that play is useful for *long-term* uncertainty reduction even if in the short term it appears otherwise, while Bass, Mahaffey, and Bonawitz (2024) show that children rely on adult beliefs about their competency to calibrate their own exploration. Colantonio, Bascandziev, Theobald, Brod, and Bonawitz (2024) present compelling modeling and experimental evidence for why and how belief revision requires executive functioning skills: not only to make predictions but also to inhibit incorrect hypotheses and switch to new ones. And Ward, Rutar, Zaadnoordijk, Poli, and Hunnius (2024) offer an impressive analysis of the fundamental questions in development, exploring what these theories might be able to say about the cognitive "starting points" and basic toolset that children enter the world with.

No special issue would be complete without a few papers that take a wider, birds-eye view of the field, and several contributions to this one filled that role nicely. Instead of focusing on the development of single humans, Koester (2024) considers the development of culture. This paper provides an absorbing explanation of cultural evolution and the emergence of cultural norms in terms of uncertainty reduction, suggesting that these adaptations exist because they enable us to better predict the environments we find ourselves in. And finally, Bramley, Zhao, Quillien, and Lucas (2024) confront one of the largest issues of all: Where do new theories come from in the first place? They posit that theory change involves an incremental search achieved through mutation and recombination of existing theories and that this can be usefully thought of as a kind of program induction.

There is much to say about all of these papers, but in the interests of space and coherence, I am going to focus less on the specifics of any one of them and more on the global themes that emerge when considering all of them as a whole. Most of the reason for this is that reading these has primarily served to crystallize my questions and clarify my uncertainties. I, therefore, think the most useful thing I can do is to simply lay these uncertainties out. I hope my thoughts are useful in identifying the areas of greatest unclarity and thus guiding the direction of future research.

2. Are these theories falsifiable? Does it matter?

As I was reading all of these papers, I kept having the same recurring worry: Are there any patterns of behavior that *cannot* be explained by these theories? This concern arises many times under many different guises, but I will focus here on two: The first is potentially more minor but also (possibly) more novel, and the second is more major.

The first derives from the fact that both predictive processing and Bayesian belief updating have uncertainty reduction at their heart: The goal of an organism is to minimize prediction error (or, as a Bayesian would put it, improve inductive accuracy). This seems entirely reasonable, but the theory permits so much latitude about the *timescale* over which that minimization can occur that it could in principle "explain" nearly anything. For instance, if we consider only the immediate moment, then an organism should never explore, only exploit—in fact, it should do its best to only sample or expose itself to data which it has already seen, because that way it is guaranteed to make the correct prediction. Conversely, if the time horizon is infinite then an organism should never *stop* exploring, because even infinitesimal prediction errors have an arbitrarily large expected value.[1]

This is in one respect nothing new: It is the classic explore/exploit dilemma. My point, though, is that *any* pattern of behavior could be made compatible with these theories via different assumptions about the expected time horizon or complexity of the world. Why do children at play construct imaginary situations for themselves, increasing uncertainty in the short term and creating arbitrary problems with no apparent point? Perhaps because doing so allows them to test and generate hypotheses, thus reducing uncertainty in the long run by helping them improve their overall conceptualization of the world (Chu & Schulz, 2020). Why do children, at other times, construct their environments so that they are not too surprising (Lillard, 2001), even though one would think that an agent motivated solely by uncertainty reduction would seek out maximum surprisal? Perhaps because they are actually motivated by *unexpected* uncertainty reduction—situations where predictions are uncertain, but not so uncertain that any outcome would be equally unsurprising (Andersen, Kiverstein, Miller, & Roepstorff, 2023). Why do children, who actively and constantly test the predictions of their theories about the world, seem predisposed to accept the norms of their social world with relatively little questioning (Schmidt, Butler, Heinz, & Tomasello, 2016)? Perhaps because of this sort of high fidelity, rapid social learning (Hoehl et al., 2019) is necessary for the promulgation of culture across generations, which itself facilitates uncertainty reduction for humans as a whole (Koester, 2024).

These are very good questions. They are also good answers! I think they are not only highly plausible but immensely appealing and elegant. But it does not escape my notice that all of the apparently inconsistent patterns of behavior raised by the questions have been answered by simply changing details of the timescale on which the uncertainty reduction operates or altering what is meant by uncertainty reduction. So what possible empirical finding would be taken as evidence *against* predictive processing or Bayesian models of cognition?

There is a second, even larger, falsifiability issue, and it comes when we consider how "optimal" should be defined. Rational constructivist approaches rely on the existence of an optimal Bayesian model explicitly, positing either that humans (including children) are well-explained by its predictions, or that deviations from it can be explained with reference to optimality: Perhaps people are solving a different computational problem than the assumed model, or have capacity limitations that require them to use approximate computations, or have prior beliefs different from what was assumed (Lieder & Griffiths, 2020; Tauber, Navarro, Perfors, & Steyvers, 2017). The predictive processing approach, too, has Bayesian optimality as a key cornerstone, positing that the agent is trying to minimize the difference between their computations and the optimal Bayesian model (Sprevak & Smith, 2024).

In both cases, a similar raft of concerns arise. Many, many choices must be made in defining any given Bayesian model. Both the prior and the likelihood are not only deeply dependent on choices made in conceptualizing the problem the learner is trying to solve but also on the type of data the learner has access to and how that data are perceived, attended to, and encoded. Moreover, every Bayesian model defines a hypothesis space, and the structure of that space has an effect on inference; as just one example, inference in a hierarchical space is very different than in a flat one (Ward et al., 2024). This means that there are many different kinds of "optimal" models. It also means that *deviations* from those models could occur for many different kinds of reasons: incorrect assumptions about the nature or quantity of the data the learner has access to, misspecification of the prior or likelihood or hypothesis space, capacity limitations resulting in poor search or other kinds of approximate inference, and so forth.

These are not new criticisms (Cao, 2020; Jones & Love, 2011), and I raise them in part because as a Bayesian modeler I have put a lot of thought into grappling with them (Perfors, 2012; Tauber et al., 2017). And in one way I do not think they are a huge problem: if we view predictive processing and rational constructivism and Bayesian models of cognition in general as scientific frameworks rather than specific theories, then they are not meant to be falsified any more than something like trigonometry is meant to be falsified. The point is not to falsify *them*; the point is to use the tools provided *by* them to construct and test falsifiable accounts of particular, specific behaviors or phenomena. For instance, within one of these frameworks, we could contrast a model that makes assumption X (about the prior or likelihood or hypothesis space or decision policy or whatever) with one that makes assumption Y instead. I think that this is the utility of *any* of these frameworks—that they allow us to precisely specify different theories within a common language and footing, and then use data to adjudicate between them. We are not falsifying (or supporting) any *framework itself*. We should use them if they are useful and not use them if they are not.

I believe that is what we should be doing, and much of the research in this area can be characterized that way; indeed, Sprevak and Smith (2024) explicitly make the same point. But the line is blurry, and we are constantly making it blurrier. If these are frameworks rather than falsifiable theories, to what extent does it even make sense to say that people "are" predictive processors or Bayesian reasoners? After all, if we can productively apply the framework no matter what the behavior, the framework itself does not have explanatory content—and this is true even if it is a tremendously useful tool for evaluating more *specific* theories about the nature of the uncertainties or hypotheses or data or constraints. Given this, to what extent does it make sense to contrast these frameworks with each other (or others) and ask questions like which approach is more or less correct? This is not a criticism of any specific people, and I include myself in it: but we blur this line, as a field, often. Probably very often. To the extent that we do, I think it is a problem.

3. Do rational constructivism and predictive processing meaningfully differ?

Somewhat ironically given the previous section, I cannot seem to figure out whether these frameworks—rational constructivism and Bayesian models of cognition more generally on the one hand, predictive processing on the other—are actually distinct in any meaningful or important way. Are there classes of behaviors or types of predictions that can be captured within one framework that cannot be explained in some analogous way within the other? Given the number of degrees of freedom and explanatory latitude that each framework offers, it may well be there there are not.[2]

And in one way that is fine—different frameworks may be more or less natural to express or evaluate different situations, even if they are isomorphic on some level—but if that is the case, it is not widely known or acknowledged. Indeed, even in this special issue, as far as I could tell, there was no evident consensus about whether or how these frameworks were distinct.

For instance, a core aspect of rational constructivism is that children are Bayesian, meaning that their behavior can be understood as rational belief updating: confronted with limited and/or ambiguous data, they combine it with their existing beliefs and expectations in a way that is consistent with Bayes' rule. Predictive processing also has Bayesian reasoning at its core, suggesting that what the brain is trying to do is minimize prediction errors (with the presumption that those errors are computed on the basis of an "optimal" Bayesian model). Active inference can also be straightforwardly captured in Bayesian terms, and Bayesian cognitive models include the machinery for handling Partially Observed Markov Decision Process (POMDPs) (Baker, Saxe, & Tenenbaum, 2011; Rafferty, Brunskill, Griffiths, & Shafto, 2016). To this extent, then, it seems to me like these frameworks are entirely compatible. Indeed, several papers in this issue appear completely consistent with both frameworks (Bass et al., 2024; Bramley et al., 2024; Lazarova et al., 2024). And one paper identifies a possible point of divergence—the assumed "starting point" within predictive processing—but then demonstrates how this apparent lack could be addressed so that the frameworks are, at least potentially, once again functionally equivalent (Ward et al., 2024).

Similar considerations arise when thinking about the fascinating contribution by Koester (2024) regarding the role of culture and norms. The idea is that human cultural learning and normativity have evolved to reduce uncertainty; they are, in essence, predictive models of the world so people who adopt them are able to reduce their uncertainty and make better predictions about the world. This is a compelling and sensible suggestion but also seems to me entirely compatible with existing functionalist (often Bayesian) explanations. For instance, there is an entire body of work explaining linguistic and cultural change as an optimization process that balances between simplicity and utility, whose quantitative basis is deeply intertwined with both information theory and Bayesian models (Kemp, Xu, & Regier, 2018).

Another paper, Andersen and Kiverstein (2024), focuses on the role of play as a difference, but the apparently opposing explanations from each framework are surprisingly similar. Both posit that play is useful because even if there are no immediate benefits (in terms of hypothesis acquisition or uncertainty reduction) there are *long-term* benefits in both. Even the claim from the predictive processing framework that play is fun because it is at the "sweet spot" of surprise seems fairly straightforward to recast in rational constructive terms: redefine the "sweet spot" not in terms of surprisal but in terms of the posterior probability.

4. Precision weighting: Whaaaat?

Aside from the different levels of analysis, the largest potential difference I can perceive between predictive processing and Bayesian models is precision weighting. To be honest, however, I cannot figure out exactly how (or if) precision weighting maps onto inference in typical Bayesian cognitive models, or even whether there is any consensus on this point.

Here is what I understand: The idea behind precision weighting is that if your prediction was wrong but it was not very precise in the first place, then you should not care much about being wrong. Precision weighting is calculated based on the (expected) inverse variance of the prediction error; it is basically the reliability of a signal in a context. My tentative conclusion is that it is already taken into account in Bayesian models (or at least *some* Bayesian models) via an appropriately defined likelihood (which according to Predictive Processing is instantiated at the neural level). For instance, Sprevak and Smith (2024) discuss a model whose hypotheses consist of Gaussians; the computations at the algorithmic level naturally separate the mean μ (which is calculated based on the weighted connections to the lower layer of neurons) and the variance σ (which is based on the lateral connections between the error units on that layer). This would correspond to a Bayesian model specified at the computational level with a likelihood represented by both μ and σ. They are thus equivalent, at least for models where the hypotheses can be decomposed in this way.[3] This Bayesian model naturally incorporates precision weighting because incorrect predictions based on likelihoods with smaller σ are penalized more relative to the same magnitude of error given a larger σ.

However, I am somewhat worried about how unconstrained precision appears to be by measurable factors. This point has been made before (Miller & Clark, 2018), and many suggestions about possible factors have been made from the variability of the environment to the importance of the outcome to the reliability of our information sources (Yon & Frith, 2021). While there is not much consensus about *which* factors matter most (and/or how or when or even how to measure or represent them), this is not itself a criticism: It is after all the nature of science, while we learn things, to not know everything already!

Nevertheless, it makes me nervous. Not only does nearly every situation faced by an organism involve most of the proposed factors, but the relationship between each factor and how precision is calculated depends on the particular model for that situation. This means that in practice one could explain nearly any pattern of results by explaining precision weighting on different levels or from different factors or over different timescales. As before, I do not think this is an in-principle terrible thing as long as it is understood that the existence (or not) of precision weighting is not itself meant to be the falsifiable claim. …But *is* that understood? We once again find ourselves balancing on that fine line between using predictive processing as a useful framework for hypothesis evaluation versus drawing conclusions as if it is itself falsifiable even though I am not sure it is.

My worries go deeper. I do not understand why an organism could not minimize their variational free energy by just having really low precision on everything. For an agent whose only goal is to minimize variational free energy, that seems the obvious solution: Setting all precision weights to zero (or, similarly, sitting in a dark quiet room) would be far easier than thinking or learning or exploring at all (Sun & Firestone, 2020). So, according to predictive processing theory, why do not organisms do this? This is not a novel question, but I am dissatisfied by most responses, which seem to me to be variations on the point that precision error is always defined relative to a model. Thus, the response goes, you should sit in a darkened room (or set your weights to zero) only if your model puts you there or tells you that you should. And a model that puts you there is not a good model because an organism with it will not be successful (they will not learn to navigate the world and will die of thirst or get eaten by tigers, and will either be apathetic or delusional until that happens).

I completely agree that such a model would be a terrible model! But what I do not understand is what, *within predictive processing theory*, tells us that it is terrible. After all, the model is great from the point of view of minimizing variational free energy; we only know it is bad because we know, as creatures in the world, that sitting around doing nothing or hallucinating is liable to get you killed. To put it another way: How does predictive processing explain *why* we have the models that we do? Why do not we adopt models that tell us to sit in a darkened room? Is predictive processing only meant to explain how we reason *given* our existing models and not where our models come from? That seems unsatisfying for a general-purpose explanatory framework of cognition.

A different response to my concern might be to say that, according to predictive processing, precision weighting reflects sensory discrimination error. Since such error is not something that an organism can control, organisms effectively do not have a choice about whether to turn off their senses or set their precision weights to zero. I find this response a bit more satisfying, but it appears to conflict with how precision is theorized about and used in the literature. For instance, Sprevak and Smith (2024) note that it is "connected to psychological features such as attention, salience, value, and uncertainty"… all of which are generally assumed to be at least *somewhat* under a person's control in real life. Moreover, active inference (which incorporates actions based on preferences) suggests to me that agents *can* have some level of control. So, if that is the case, I repeat: Why should they not choose to simply "not care" about anything[4] via a precision weight of zero? In the real world, one would imagine that learners would want *increased* precision in areas they care about, but does not predictive processing suggest the opposite, since increasing precision would actually make the free energy larger, all other factors remaining equal?

Relatedly, it is not clear to me how (or if) predictive processing provides a mechanism by which an organism could learn about or access their own precision. The role that precision weighting plays in guiding exploration (and examples like the car in fog given by Ward et al., 2024) all strongly imply to me that predictive processing *presumes* that agents have some sort of access to their own precision… but I cannot find a mechanism *within the theory* explaining how precision weighting might change over time or be represented or learned. The only mechanism I can see for how precision weighting affects anything is via the implementation-level processes involving the error rates on neurons—which strongly implies that there is *no* learning or conscious access. But besides being at odds with other presumptions elsewhere, this also seems empirically incorrect. We *do* at least sometimes have access to our own sensory prediction error, though probably with some level of error. Moreover, sensory prediction error changes constantly over development or as a result of injury or training, and we require time to adjust to those changes, indicating that learning does occur. So how are these things explained within predictive processing?

5. I should put a summary here

I feel like all I have done is ask a bunch of questions, which makes writing a summary difficult. I actually am a fan of all of the theoretical frameworks under discussion, and I think this special issue brought together a great set of papers. I have nitpicks and disagreements with (parts of) many of them, but that is absolutely normal, and they all contribute to our understanding of cognitive development. I did not write about their greatness, though, because that is a bit boring.

Rather—to get meta for a moment—I thought these questions were the best way to maximize the value of my commentary: They have the highest probability of leading to the most uncertainty reduction for our field in the long term. They *also* have the highest probability of making me look silly. In writing this, I pushed the boundaries of my own knowledge; it is up to you to decide whether this is at the "sweet spot" of maximum information gain or utterly useless or somewhere in the middle. So I will conclude by begging your forbearance. Please take this commentary in the spirit it was written: As scientific play, throwing some stuff out there in the hopes that it leads somewhere for someone. If not, oh well! Do not judge the papers based on the high-entropy nature of anything I said here, and thanks for bearing with me.

Notes

1 Even there, there is nuance, of course: if the actual complexity of the world is low enough that the organism is able to fully explore it all, then it should stop. But no organism knows in advance what the actual complexity of the world is, and any explanation of exploration/exploitation still must somehow take that complexity into account.

2 One obvious difference is that predictive processing account extends through all three of Marr's levels while Bayesian models apply only at the computational level. My question is whether this leads to any differences *at the computational level*. Are there any phenomena at that level that are explained by the predictive processing framework in a qualitatively distinct way from the Bayesian framework, due to the lower levels of analysis of predictive processing?

3 There are many probability distributions other than Gaussians that have a parameter that is conceptually like the variance, but I am not clear whether predictive processing

includes models where the algorithmic-level computations have distributions with those forms. My sense is that if it does, this is not at all standard. If this is correct it suggests to me that Bayesian models, which can include any mathematically definable likelihood, are a superset of predictive processing models, at least at the computational level.

4 Obviously the answer is "because then the organism would get eaten by a tiger." But I am asking about what the answer is from within a predictive processing perspective, which states that the entire goal is to minimize variational free energy. If an easy way to do this is something that no organisms do, and if they did they would be eaten by tigers or sit motionless till they starved to death, then perhaps this is not the entire goal.

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